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Aspects of D-branes dynamics on orbifolds

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Abstract

We discuss D-brane dynamics in orbifold compactifications of type II superstring theory. We compute the interaction potential between two D-branes moving with constant velocities and give a field theory interpretation of it in the large distance limit.

Talk presented by Claudio A. Scrucca

1 Introduction and summary

We study various D-brane configurations in orbifold compactifications which are D-particles from the 4-dimensional space-time point of view, but can have extension in the compact directions. More precisely, the cases of the 0-brane of type IIA and a particular 3-brane of type IIB, turn out to be particularly interesting.

The dynamics of these D-branes is determined by a one loop amplitude which can be conveniently evaluated in the boundary state formalism [1, 2]. In particular, one can compute the force between two D-branes moving with constant velocity, extending Bachas' result [3] to compactifications breaking some supersymmetry [4]. Analyzing the large distance behavior of the interaction and its velocity dependence, it is possible to read the charges with respect to the massless fields, and relate the various D-brane configurations to known solutions of the 4-dimensional low energy effective supergravity.

Finally, we discuss the emission of massless particle from two D-branes in interaction [5]. We compute the average energy which is radiated when two D-branes pass each other.

2 Interactions on orbifolds

Consider two D-branes moving with velocities $V_1 = \tanh v_1$, $V_2 = \tanh v_2$ (say along 1) and transverse positions $\vec{Y}_1$, $\vec{Y}_2$ (along 2,3). The potential between these two D-branes is given by the cylinder vacuum amplitude and can be thought either as the Casimir energy stemming from open string vacuum fluctuations or as the interaction energy related to the exchange closed strings between the two branes. The amplitude in the closed string channel

$$\mathcal{A} = \int_0^\infty dl \sum_s < B, V_1, \vec{Y}_1 | e^{-lH} | B, V_2, \vec{Y}_2 >_s \qquad (1)$$

is just a tree level propagation between the two boundary states, which are defined to implement the boundary conditions defining the branes. There are two sectors, RR and NSNS, corresponding to periodicity and antiperiodicity of the fermionic fields around the cylinder, and after the GSO projection there are four spin structures, R± and NS±, corresponding to all the possible periodicities of the fermions on the covering torus.

In the static case, one has Neumann b.c. in time and Dirichlet b.c. in space. The velocity twists the 0-1 directions and gives them rotated b.c. The moving boundary state is most simply obtained by boosting the static one with a negative rapidity $v = v_1 - v_2$ [6].

$$|B, V, \vec{Y}>= e^{-ivJ^{01}}|B, \vec{Y}> \ .$$

In the large distance limit $b \to \infty$ only world-sheets with $l \to \infty$ will contribute, and momentum or winding in the compact directions can be safely neglected since they correspond to massive subleading components.

The moving boundary states

$$|B, V_1, \vec{Y}_1>= \int \frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{Y}_1}|B, V_1> \otimes |k_B> \ , \quad |B, V_2, \vec{Y}_2>= \int \frac{d^3\vec{q}}{(2\pi)^3} e^{i\vec{q}\cdot\vec{Y}_2}|B, V_2> \otimes |q_B> \ ,$$

can carry only space-time momentum in the boosted combinations $k^\mu_B = (\sinh v_1 k^1, \cosh v_1 k^1, \vec{k}_T)$ and $q^\mu_B = (\sinh v_2 q^1, \cosh v_2 q^1, \vec{q}_T)$. Notice that because of their non zero velocity, the branes can also transfer energy, and not only momentum as in the static case.

Integrating over the bosonic zero modes and taking into account momentum conservation ($k^\mu_B = q^\mu_B$), the amplitude factorizes into a bosonic and a fermionic partition functions:

$$\mathcal{A} = \frac{1}{\sinh v} \int_0^\infty dl \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q_B^2}{2}} \sum_s Z_B Z_F^s = \frac{1}{\sinh v} \int_0^\infty \frac{dl}{2\pi l} e^{-\frac{b^2}{2l}} \sum_s Z_B Z_F^s \tag{2}$$

with $Z_{B,F} =< B, V_1|e^{-lH}|B, V_2>^s_{B,F}$. From now on, $X^\mu \equiv X^\mu_{osc}$.

It will prove very convenient to group the fields into pairs,

$$\begin{aligned} X^\pm = X^0 \pm X^1 &\to \alpha_n, \beta_n = a^0_n \pm a^1_n \ , \\ X^i, X^{i*} = X^i \pm iX^{i+1} &\to \beta^i_n, \beta^{i*}_n = a^i_n \pm ia^{i+1}_n \ , \quad i = 2, 4, 6, 8 \ , \\ \chi^{A,B} = \psi^0 \pm \psi^1 &\to \chi^{A,B}_n = \psi^0_n \pm \psi^1_n \ , \\ \chi^i, \chi^{i*} = \psi^i \pm i\psi^{i+1} &\to \chi^i_n, \chi^{i*}_n = \psi^i_n \pm i\psi^{i+1}_n \ , \quad i = 2, 4, 6, 8 \ , \end{aligned}$$

with the commutation relations $[\alpha_m, \beta_{-n}] = -2\delta_{mn}$, $[\beta^i_m, \beta^{i*}_{-n}] = 2\delta_{mn}$, $\{\chi^A_m, \chi^B_{-n}\} = -2\delta_{mn}$, $\{\chi^i_m, \chi^{i*}_n\} = 2\delta_{mn}$. For the RR zero modes, which satisfy a Clifford algebra and are thus proportional to Γ-matrices, $\psi^\mu_o = i\Gamma^\mu/\sqrt{2}$, $\tilde{\psi}^\mu_o = i\tilde{\Gamma}^\mu/\sqrt{2}$, on can construct similarly the creation-annihilation operators

$$a, a^* = \frac{1}{2}(\Gamma^0 \pm \Gamma^1) \ , \quad b^i, b^{i*} = \frac{1}{2}(-i\Gamma^i \pm \Gamma^{i+1}) \ ,$$

and similarly for tilded operators, satisfying the usual algebra $\{a, a^*\} = \{b^i, b^{i*}\} = 1$.

In this way, any rotation or boost will reduce to a simple phase transformation on the modes. In fact, for an orbifold rotation ($g_a = e^{2\pi i z_a}$) one has

$$\begin{aligned} &\beta^a_n \to g_a \beta^a_n \ , \quad \chi^a_n \to g_a \chi^a_n \ , \quad b^a \to g_a b^a \ , \\ &\beta^{a*}_n \to g^*_a \beta^{a*}_n \ , \quad \chi^{a*}_n \to g^*_a \chi^{a*}_n \ , \quad b^{a*} \to g^*_a b^{a*} \ . \end{aligned} \tag{3}$$

whereas for a boost of rapidity v,

$$\begin{aligned}&\alpha_n \to e^{-v}\alpha_n \ , \ \ \chi^A_n \to e^{-v}\chi^A_n \ , \ \ a \to e^{-v}a \ , \\ &\beta_n \to e^{v}\beta_n \ , \ \ \chi^B_n \to e^{v}\chi^B_n \ , \ \ a^* \to e^{v}a^* \ .\end{aligned} \tag{4}$$

The boundary state which solves the b.c. can be factorized into a bosonic and a fermionic parts; the latter can be further splitted into zero mode and oscillator parts, and finally

$$|B>=|B>_B \otimes|B_o>_F \otimes|B_{osc}>_F \ \ .$$

2.1 Orbifold construction

An orbifold compactification can be obtained by identifying points in the compact part of space-time which are connected by discrete rotations $g = e^{2\pi i \sum_a z_a J_{aa+1}}$ on some of the compact pairs X^a,χ^a, a=4,6,8. In order to preserve at least one supersymmetry, one has to impose $\sum_a z_a = 0$.

We will consider three case: toroidal compactification on T_6 ($N = 8$ SUSY, $z_4 = z_6 = z_8 = 0$) and orbifold compactification on $T_2 \otimes T_4/Z_2$ ($N = 4$ SUSY, $z_4 = -z_6 = \frac{1}{2}$, $z_8 = 0$) and T_6/Z_3 ($N = 2$ SUSY, $z_4, z_6 = \frac{1}{3}, \frac{2}{3}$, $z_8 = -z_4 - z_6$).

The spectrum of the theory now contains additional twisted sectors, in which periodicity is achieved only up to an element of the quotient group Z_N. These twisted states exist at fixed points of the orbifold, and can thus occur only for 0-branes localized at one of the fixed points. We will not discuss this case here (see [4]).

Finally, in all sectors, one has to project onto invariant states to get the physical spectrum of the theory which is invariant under orbifold rotations. In particular, the physical boundary state is given by the projection $|B_{phys}>= 1/N\sum_k |B, g^k>$) in terms of the twisted boundary states $|B, g^k>= g^k|B>$.

2.2 0-brane

Consider first the static case, for which the b.c. are Neumann for time and Dirichlet for all other directions (i=2,4,6,8 and a=2,4,6). The bosonic b.c. translate into the following equations

$$\begin{aligned}&(\alpha_n + \tilde{\beta}_{-n})|B>_B= 0 \ , \ \ (\beta_n + \tilde{\alpha}_{-n})|B>_B= 0 \ , \\ &(\beta^i_n - \tilde{\beta}^i_{-n})|B>_B= 0 \ , \ \ (\beta^{i*}_n - \tilde{\beta}^{i*}_{-n})|B>_B= 0 \ ,\end{aligned}$$

For the fermions, one has integer or half-integer moding in the RR and NSNS sectors respectively.

$$\begin{aligned}&(\chi^A_n + i\eta\tilde{\chi}^B_{-n})|B_{osc},\eta>_F= 0 \ , \ \ (\chi^B_n + i\eta\tilde{\chi}^A_{-n})|B_{osc},\eta>_F= 0 \ , \\ &(\chi^i_n - i\eta\tilde{\chi}^i_{-n})|B_{osc},\eta>_F= 0 \ , \ \ (\chi^{i*}_n - i\eta\tilde{\chi}^{i*}_{-n})|B_{osc},\eta>_F= 0 \ . \\ &(a + i\eta\tilde{a}^*)|B_o,\eta>_F= 0 \ , \ \ (a^* + i\eta\tilde{a})|B_o,\eta>_F= 0 \ , \\ &(b^i - i\eta\tilde{b}^i)|B_o,\eta>_F= 0 \ , \ \ (b^{i*} - i\eta\tilde{b}^{i*})|B_o,\eta>_F= 0 \ .\end{aligned}$$

Here $\eta = \pm 1$ has been introduced to deal later on with the GSO projection.

The boundary state solving these b.c. is easily constructed as a Bogolubov transformation from a spinor vacuum $|0> \otimes|\tilde{0}>$ defined such that $a|0>= \tilde{a}|\tilde{0}>= b^i|0>= \tilde{b}^{i*}|\tilde{0}>= 0$. After applying the boost eq. (4), under which the spinor vacuum picks up an imaginary phase,

$|0> \otimes|\tilde{0}> \to e^{-v}|0> \otimes|\tilde{0}>$, the result is

$$
\begin{aligned}
|B,V>_B &= \exp\left\{\frac{1}{2}\sum_{n>0}(e^{-2v}\alpha_{-n}\tilde{\alpha}_{-n} + e^{2v}\beta_{-n}\tilde{\beta}_{-n} + \beta^i_{-n}\tilde{\beta}^{i*}_{-n} + \beta^{i*}_{-n}\tilde{\beta}^i_{-n})\right\}|0> , \\
|B_{osc},V,\eta>_F &= \exp\left\{\frac{i\eta}{2}\sum_{n>0}(e^{-2v}\chi^A_{-n}\tilde{\chi}^A_{-n} + e^{2v}\chi^B_{-n}\tilde{\chi}^B_{-n} - \chi^i_{-n}\tilde{\chi}^{i*}_{-n} - \chi^{i*}_{-n}\tilde{\chi}^i_{-n})\right\}|0> , \qquad (5) \\
|B_o,V,\eta>_{RR} &= e^{-v}\exp\left\{-i\eta(e^{2v}a^*\tilde{a}^* - b^{i*}\tilde{b}^i)\right\}|0> \otimes|\tilde{0}> \ .
\end{aligned}
$$

The complete boosted boundary state is already invariant under orbifold rotations eq. (3). This comes from the fact that the Z_N action rotates pairs of fields with the same b.c. and is thus irrelevant.

In both sectors, the fermion number operator reverses the sign of the parameter η, that is $(-1)^F|B,V,\eta>= -|B,V,-\eta>$, and the GSO-projected boundary state is given by the difference $|B,V>=\frac{1}{2}(|B,V,+> - |B,V,->)$. There will thus be two kinds of contributions in the amplitude for each sector: the one with equal η-parameters for both boundary states and the one with opposite η-parameters, giving finally four spin structures.

The partition function can then be computed carrying out some simple oscillator algebra; the ghosts cancel one untwisted pair, say 2-3, and the result is the product of the contributions of the 0-1 pair and the 3 compact pairs.

After the GSO projection, only the three even spin structures R+ and NS$\pm$ contribute, and the total bosonic (zero-point energy $q^{-\frac{2}{3}}$) and fermionic (zero-point energy $q^{-\frac{1}{3}}$ for NSNS and $q^{\frac{2}{3}}$ for RR) partition functions are ($q=e^{-2\pi l}$)

$$
Z_B = 16\pi^3 i \sinh v q^{\frac{1}{3}} f(q^2)^4 \frac{1}{\vartheta_1(i\frac{v}{\pi}|2il)\vartheta_1'(0|2il)^3} , \qquad (6)
$$

$$
\begin{aligned}
Z_F &= q^{-\frac{1}{3}} f(q^2)^{-4}\left\{\vartheta_2(i\frac{v}{\pi}|2il)\vartheta_2(0|2il)^3 - \vartheta_3(i\frac{v}{\pi}|2il)\vartheta_3(0|2il)^3 + \vartheta_4(i\frac{v}{\pi}|2il)\vartheta_4(0|2il)^3\right\} \\
&\sim V^4 , \qquad (7)
\end{aligned}
$$

corresponding to the usual cancellation of the force between two BPS states [7, 3]. Thus, for the 0-brane we get the same result as the uncompactified theory for every compactification scheme.

2.3 3-brane

Let us now consider a particular 3-brane configuration. In the static case, we take Neumann b.c. for time, Dirichlet b.c. for space and mixed b.c. for each pair of compact directions, say Neumann for the a directions and Dirichlet for the a+1 directions.

The new b.c. for the compact directions are

$$
\begin{aligned}
&(\beta^a_n + \tilde{\beta}^{a*}_{-n})|B>_B = 0 , \quad (\beta^{a*}_n + \tilde{\beta}^a_{-n})|B>_B = 0 , \\
&(\chi^a_n + i\eta\tilde{\chi}^{a*}_{-n})|B_{osc},\eta>_F = 0 , \quad (\chi^{a*}_n + i\eta\tilde{\chi}^a_{-n})|B_{osc},\eta>_F = 0 , \\
&(b^a + i\eta\tilde{b}^{a*})|B_o,\eta>_F = 0 , \quad (b^{a*} + i\eta\tilde{b}^a)|B_o,\eta>_F = 0 .
\end{aligned}
$$

Defining a new spinor vacuum $|0> \otimes|\tilde{0}>$ such that $b^a|0>=\tilde{b}^a|\tilde{0}>=0$ the compact part of the boundary state is constructed in the same way as before. In this case, however, the boundary state is not invariant under orbifold rotations, under which the modes of the fields transform as in eq. (3) and the new spinor vacuum as $|0> \otimes|\tilde{0}> \to g_a|0> \otimes|\tilde{0}>$. This was expected since a Z_N rotation now mixes two directions with different b.c, and thus the corresponding closed string state does not need to be invariant under Z_N rotations. One finds for the compact part of the twisted boundary state

$$
\begin{aligned}
&|B,V,g_a>_B = \exp\left\{-\frac{1}{2}\sum_{n>0}(g_a^2\beta^a_{-n}\tilde\beta^a_{-n} + g_a^{*2}\beta^{a*}_{-n}\tilde\beta^{a*}_{-n})\right\}|0>\ , \\
&|B_{osc},V,g_a,\eta>_F = \exp\left\{\frac{i\eta}{2}\sum_{n>0}(g_a^2\chi^a_{-n}\tilde\chi^a_{-n} + g_a^{*2}\chi^{a*}_{-n}\tilde\chi^{a*}_{-n})\right\}|0>\ , \qquad (8)\\
&|B_o,V,g_a,\eta>_{RR} = g_a\exp\left\{-i\eta g_a^{*2}b^{a*}\tilde b^{a*}\right\}|0>\otimes|\tilde 0>\ .
\end{aligned}
$$

After the GSO projection, the total partition functions for a given relative angle w_a are

$$
Z_B = 16i\sinh v q^{\frac{1}{3}} f(q^2)^4 \frac{1}{\vartheta_1(i\frac{v}{\pi}|2il)}\prod_a \frac{\sin\pi w_a}{\vartheta_1(w_a|2il)}\ , \qquad (9)
$$

$$
\begin{aligned}
Z_F &= q^{-\frac{1}{3}} f(q^2)^{-4}\left\{\vartheta_2(i\frac{v}{\pi}|2il)\prod_a\vartheta_2(w_a|2il)\right.\\
&\qquad\left. -\vartheta_3(i\frac{v}{\pi}|2il)\prod_a\vartheta_3(w_a|2il) + \vartheta_4(i\frac{v}{\pi}|2il)\prod_a\vartheta_4(w_a|2il)\right\}\\
&\sim\left\{\begin{array}{ll} V^4 \ , & w_a = 0 \\ V^2\ , & w_a \neq 0\end{array}\right. . \qquad (10)
\end{aligned}
$$

Recall that to obtain the invariant amplitude, one has to average over all possible angles w_a.

3 Large distance limit

In the large distance limit $l\to\infty$, explicit results with exact dependence on the rapidity can be obtained and compared to a field theory computation. One finds the following behaviors:

0-brane

$$
\mathcal{A}\sim 4\cosh v - \cosh 2v - 3 \sim V^4\ . \qquad (11)
$$

3-brane

$$
\begin{aligned}
&\mathcal{A}(w_a)\sim 4\prod_a\cos\pi w_a\cosh v - \cosh 2v - \sum_a\cos 2\pi w_a\ ,\\
&\mathcal{A}\sim\left\{\begin{array}{ll}\cosh v - \cosh 2v\sim V^2\ , & T_6/Z_3\\ 4\cosh v - \cosh 2v - 3\sim V^4\ , & T_2\otimes T_4/Z_2\ ,\ T_6\end{array}\right. . \qquad (12)
\end{aligned}
$$

The additional twisted sectors can be analyzed similarly, and one finds $\mathcal{A}\sim\cosh v - 1\sim V^2$.

In the low energy effective supergravity field theories, the possible contributions to the scattering amplitude in the eikonal approximation come from vector exchange in the RR sector and dilaton and graviton exchange in the NSNS sector. The respective contributions have a peculiar dependence on the rapidity reflecting the tensorial nature and are:

$$
\mathcal{A}^{NS}_\phi\sim -a^2\ ,\ \ \mathcal{A}^R_{V_\mu}\sim e^2\cosh v\ ,\ \ \mathcal{A}^{NS}_{g_{\mu\nu}}\sim -M^2\cosh 2v\ . \qquad (13)
$$

Thus, the interpretation of the behaviors found in the various sectors and for the various brane configurations we have considered, is the following:

$$
\begin{aligned}
4\cosh v - \cosh 2v - 3 &\Leftrightarrow N=8 \text{ Grav. multiplet}\ ,\\
\cosh v - \cosh 2v &\Leftrightarrow N=2 \text{ Grav. multiplet}\ ,\\
\cosh v - 1 &\Leftrightarrow \text{Vec. multiplet}\ .
\end{aligned}
$$

The patterns of cancellation suggest that all the D-brane configurations that we have considered correspond to extremal 0-brane solutions of the low energy 4-dimensional supergravity, possibly coupling to additional twisted vector multiplets; the 3-brane configuration on the Z_3 orbifold seems to be an exception since it does not couple to scalars, and should thus correspond to a Reissner-Nordström extremal black hole.

Finally, notice that V^2 terms in the effective action give a non flat metric to the moduli space. Since in the dual open string channel a constant velocity V corresponds by T-duality to a constant electric field E, V^2 terms correspond to a renormalization of the Maxwell term E^2. It is well known that this can not happen for maximally supersymmetric theories; the V^2 behavior is thus forbidden for $N = 8$ compactifications, but generically allowed for compactifications breaking some supersymmetry, $N < 8$. Our results are compatible with this and show that V^2 terms do indeed appear in some cases.

4 Particle emission

For non zero relative velocity between the branes, particle emission is kinematically allowed even in the eikonal approximation. The corresponding amplitude can be computed inserting the appropriate vertex operator in the matrix element (1); we have done it for various NSNS particle emission [5]. At large inter-brane distances, emission occurs only for the scalars of the $N = 8$ gravitational multiplet (hence there is no scalar emission in the case of the 3-brane which couples to the $N = 2$ gravitational multiplet) and for the 4-dimensional graviton. One can compute the average energy radiated through graviton emission, finding

$$< p >\sim g_s^2 l_s^2 \frac{V^{1+2n}}{b^3} \ , \qquad (14)$$

with $n = 2, 4$ depending on the amount of preserved supersymmetry.

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